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2022

Time : 3 hours

Full Marks : 70

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer from both the Sections as directed.

Section – A
(Long-answer Type Questions)

Answer any **four** questions of the following : $10\times4 = 40$

18

48

1. Define homogeneous function of degree n. State and prove Euler's theorem on partial differentiation.

5

2. If $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$, prove that $a^2y^3\frac{d^2y}{dx^2} + b^4 = 0$.

3. Find maxima and minima of the function

$$x^3 + y^3 - 12x - 3y + 15.$$

4. Apply Maclaurin's series to prove the expansion

$$\log(1 + \tan x) = x - \frac{x^2}{2!} + \frac{4x^3}{3!} \cdots \text{ to } \infty.$$

5. If $y = (x^2 - 1)$, then prove that

$$(x^2 - 1)y_{n+2} + 2xy_{n+1} - n(n + 1)\, y_n = 0$$

6. Find length of the loop of the curve

$$3ay^2 = x(x - a)^2$$

7. Solve the equation : $(1 + y^2)\dfrac{dx}{dy} + x = \tan^{-1} y$

8. Find $\dfrac{d^2y}{dx^2}$, if $x = a\cos\theta$, $y = b\sin\theta$.

Section – B

(Short-answer Type Questions)

9. Answer **all** questions of the following : $3 \times 10 = 30$

(a) Solve the equation $\dfrac{dy}{dx} = \dfrac{1+y^2}{1+x^2}$.

(b) Find y_3 if $y = x^2 \log x$.

(c) If $y = \sin^{-1}x$, prove that $(1 + x^2)y_2 - xy_1 = 0$.

(d) Find extreme value of the function

$$f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$$

(e) Find the area bounded by the curve $y = 2 - x^2$ and the straight line $y = x$.

(f) Expand $\log(1 + x)$ by Maclaurin's theorem.

(g) Solve the differential equation,

$$x\frac{dy}{dx} - 3y = x^2$$

(h) Find the volume in the first octant bounded by the planes $x + z = 1$ and $y + 2z = 2$.

(i) Find the area enclosed by Lemniscate

$$r^2 = 2a^2\cos\theta$$

(j) Apply Maclaurin's series to prove the following expansion !

$$\sec x = 1 + \frac{x^2}{2!} + \frac{5x^4}{4!} + \cdots\cdots \infty$$