

2022

Time : 3 hours

Full Marks : 70

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer from both the Sections as directed.

**Section – A**

**(Long-answer Type Questions)**

Answer any four questions of the following :

10×4 = 40

1. Define homogeneous function of degree n. State and prove Euler's theorem on partial differentiation.

2. If  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , prove that  $a^2 y^3 \frac{d^2 y}{dx^2} + b^4 = 0$ .

3. Find maxima and minima of the function

$$x^3 + y^3 - 12x - 3y + 15.$$

4. Apply Maclaurin's series to prove the expansion

5  $\log(1 + \tan x) = x - \frac{x^2}{2!} + \frac{4x^3}{3!} \dots \text{to } \infty.$

5. If  $y = (x^2 - 1)^n$ , then prove that

$$(x^2 - 1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$$

6. Find length of the loop of the curve

$$3ay^2 = x(x - a)^2$$

7. Solve the equation:  $(1 + y^2) \frac{dx}{dy} + x = \tan^{-1} y$

10 8. Find  $\frac{d^2y}{dx^2}$ , if  $x = a \cos \theta$ ,  $y = b \sin \theta$ .

### Section - B

#### (Short-answer Type Questions)

9. Answer all questions of the following:  $3 \times 10 = 30$

3 (a) Solve the equation  $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ .

5 (b) Find  $y_3$  if  $y = x^2 \log x$ .

12 (c) If  $y = \sin^{-1} x$ , prove that  $(1 + x^2)y_2 - xy_1 = 0$ .

(d) Find extreme value of the function

1

$$f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$$

(e) Find the area bounded by the curve  $y =$

1

$2 - x^2$  and the straight line  $y = x$ .

(f) Expand  $\log(1 + x)$  by Maclaurin's theorem.

2

(g) Solve the differential equation,

3

$$x \frac{dy}{dx} - 3y = x^2$$

$$\sqrt{1+x^2} = 2 \cdot 2\sqrt{1+x^2}$$

(h) Find the volume in the first octant bounded by the planes  $x + z = 1$  and  $y + 2z = 2$ .

(i) Find the area enclosed by Lemniscate

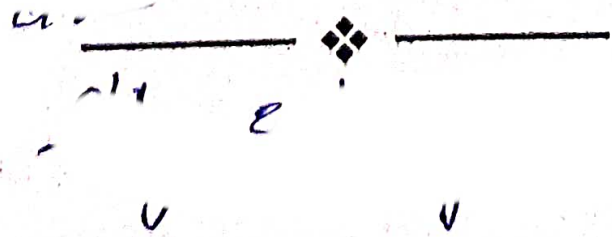
1

$$r^2 = 2a^2 \cos \theta$$

(j) Apply Maclaurin's series to prove the following expansion !

2

$$\sec x = 1 + \frac{x^2}{2!} + \frac{5x^4}{4!} + \dots \infty$$



2 | + n^2